IFT-04-00
TOKUSHIMA Report
(hep-ph/0004223)

Optimal-Observable Analysis of the Angular and Energy Distributions for Top-Quark Decay Products at Polarized Linear Colliders

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ABSTRACT

An optimal-observable analysis of the angular and energy distributions of the leptons and bottom quarks in the process $e^+e^- \to t\bar{t} \to \ell^\pm/\overset{(-)}{b}\cdots$ has been performed in order to measure the most general top-quark couplings to gauge bosons at polarized linear colliders. The optimal beam polarization for determination of each coupling has been found. A very sensitive test of CP violation in $t\bar{t}$ production and decay has been proposed.

PACS: 13.65.+i

Keywords: top quark, CP violation, anomalous top-quark interactions

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1. Introduction

In spite of the fact that the top quark has been discovered already several years ago [1] its interactions are still very weakly constrained. It remains an open question if top-quark couplings obey the Standard Model (SM) scheme of the electroweak forces or there exists a contribution from physics beyond the SM. We could interpret the great success of the 1-loop precision tests of the SM as a strong indication that the third generation also obeys the SM scheme. However, an independent and direct measurement of the top-quark couplings is definitely necessary before drawing any definite conclusion concerning non-standard physics.

Over the past several years there was a substantial effort devoted to a possibility of determining top-quark couplings through measurements performed at the open top region[♯1] of future e^+e^- linear colliders [3]–[6]. The existing studies focused mainly on tests of *CP* violation in top-quark interactions. In this article we will construct some new tools which could help to measure both *CP violating and CP conserving* top-quark couplings at linear colliders and therefore reveal the structure of fundamental interactions beyond the SM.

The top quark decays immediately after being produced and its huge mass $m_t \simeq 174$ GeV leads to a decay width Γ_t much larger than $\Lambda_{\rm QCD}$. Therefore the decay process is not influenced by any fragmentation effects [7] and decay products will provide useful information on top-quark properties. Here we will consider distributions of either $\ell^\pm$ in the inclusive process $e^+e^- \to t\bar{t} \to \ell^\pm \cdots$ or bottom quarks from $e^+e^- \to t\bar{t} \to \overset{(-)}{b} \cdots$. It turns out that the analysis of the leptonic and b-quark final states is similar and could be presented simultaneously.

This paper is organized as follows. First in sec.2 we describe the basic framework of our analysis, and then show the angular and energy distributions of the lepton and b-quark in sec.3. In sec.4, after briefly reviewing the optimal-observable procedure [8], we estimate to what precision all the non-standard parameters can be measured or constrained adjusting the initial beam polarizations. Finally, we

[♯1] Recently an interesting and complementary analysis by Jezabek, Nagano and Sumino has been published [2] where the authors discussed possibility of determining *CP*-violating production form factors at the $t\bar{t}$ threshold region.

summarize our results in sec.5. In the appendix we collect several functions used in the main text for completeness, though some of them could also be found in our previous papers [4, 5].

2. Framework and Formalism

We parameterize $t\bar{t}$ couplings to the photon and the Z boson in the following way

$$\Gamma^{\mu}_{vt\bar{t}} = \frac{g}{2}\,\bar{u}(p_t)\left[\gamma^{\mu}\{A_v+\delta A_v-(B_v+\delta B_v)\gamma_5\}+\frac{(p_t-p_{\bar{t}})^{\mu}}{2m_t}(\delta C_v-\delta D_v\gamma_5)\right]v(p_{\bar{t}}), \quad (2.1)$$

where g denotes the $SU(2)$ gauge coupling constant, $v=\gamma, Z$, and

$$A_{\gamma}=\frac{4}{3}\sin\theta_W,\quad B_{\gamma}=0,\quad A_Z=\frac{1}{2\cos\theta_W}\Big(1-\frac{8}{3}\sin^2\theta_W\Big),\quad B_Z=\frac{1}{2\cos\theta_W}$$

denote the SM contributions to the vertices. Among the above non-SM form factors, δA_v, δB_v, δC_v describe CP-conserving while δD_v parameterizes CP-violating interactions. Similarly, we adopt the following parameterization of the Wtb vertex suitable for the t and $\bar{t}$ decays:

$$\begin{aligned}
\Gamma^{\mu}_{Wtb} &= -\frac{g}{\sqrt{2}}V_{tb}\,\bar{u}(p_b)\Big[\gamma^{\mu}(f_1^L P_L+f_1^R P_R)-\frac{i\sigma^{\mu\nu}k_{\nu}}{M_W}(f_2^L P_L+f_2^R P_R)\Big]u(p_t),\\
\bar{\Gamma}^{\mu}_{Wtb} &= -\frac{g}{\sqrt{2}}V^*_{tb}\,\bar{v}(p_{\bar{t}})\Big[\gamma^{\mu}(\bar{f}_1^L P_L+\bar{f}_1^R P_R)-\frac{i\sigma^{\mu\nu}k_{\nu}}{M_W}(\bar{f}_2^L P_L+\bar{f}_2^R P_R)\Big]v(p_{\bar{b}}), \quad (2.2)
\end{aligned}$$

where $P_{L/R}=(1\mp\gamma_5)/2$, V_{tb} is the (tb) element of the Kobayashi-Maskawa matrix and k is the momentum of W. In the SM $f_1^L=\bar{f}_1^L=1$ and all the other form factors vanish. On the other hand, it is assumed here that interactions of leptons with gauge bosons are properly described by the SM. Throughout the calculations all fermions except the top quark are considered as massless. We also neglect terms quadratic in the non-standard form factors.

Using the technique developed by Kawasaki, Shirafuji and Tsai [9] one can derive the following formula for the inclusive distributions of the top-quark decay product f in the process $e^+e^-\to t\bar{t}\to f+\cdots$ [4]:

$$\frac{d^3\sigma}{d\boldsymbol{p}_f/(2p_f^0)}(e^+e^-\to f+\cdots)=4\int d\Omega_t\frac{d\sigma}{d\Omega_t}(n,0)\frac{1}{\Gamma_t}\frac{d^3\Gamma_f}{d\boldsymbol{p}_f/(2p_f^0)}(t\to f+\cdots), \quad (2.3)$$

where Γ_t is the total top-quark decay width and $d^3\Gamma_f$ is the differential decay rate for the process considered. $d\sigma(n,0)/d\Omega_t$ is obtained from the angular distribution of $t\bar{t}$ with spins s_+ and s_- in $e^+e^- \to t\bar{t}$, $d\sigma(s_+,s_-)/d\Omega_t$, by the following replacement:

$$s_{+\mu} \to n^f_\mu = -\Big[\, g_{\mu\nu} - \frac{p_{t\mu}p_{t\nu}}{m_t^2}\,\Big] \frac{\sum \int d\Phi\; \bar{B}\Lambda_+\gamma_5\gamma^\nu B}{\sum \int d\Phi\; \bar{B}\Lambda_+ B}, \quad s_{-\mu} \to 0, \tag{2.4}$$

where the matrix element for $t(s_+) \to f + \cdots$ was expressed as $\bar{B}u_t(p_t,s_+)$, $\Lambda_+ \equiv \not{p}_t + m_t$, $d\Phi$ is the relevant final-state phase-space element and $\sum$ denotes the appropriate spin summation.

3. Angular/Energy Distributions

In this section we present $d^2\sigma/dx_f d\cos\theta_f$ for the top-quark decay product $f(= \ell^\pm/\overset{(-)}{b})$, where x_f denotes the normalized energy of f defined in terms of its energy E_f and the top-quark velocity $\beta(\equiv \sqrt{1-4m_t^2/s})$ as

$$x_f \equiv \frac{2E_f}{m_t}\sqrt{\frac{1-\beta}{1+\beta}}$$

and θ_f is the angle between the e^- beam direction and the f momentum, all in the e^+e^- CM frame.

Direct calculations performed in presence of the general decay vertex (2.2) lead to the following result for the n^f_μ vector defined in eq.(2.4):

$$n^f_\mu = \alpha^f \left(g_{\mu\nu} - \frac{p_{t\mu}p_{t\nu}}{m_t^2} \right) \frac{m_t}{p_t p_f} p_f^\nu \tag{3.1}$$

where for a given final state f, α^f is a calculable depolarization factor

$$\alpha^f = \begin{cases} 1 & \text{for} \;\; f = \ell^+ \\ \dfrac{2r-1}{2r+1}\Big[\, 1 + \dfrac{8\sqrt{r}(1-r)}{(2r-1)(2r+1)} \mathrm{Re}(f_2^R)\,\Big] & \text{for} \;\; f = b \end{cases} \tag{3.2}$$

with $r \equiv (M_W/m_t)^2$. Similarly we have $\alpha^{\bar{f}} = -\alpha^f$ with replacement $f_2^R \to \bar{f}_2^L$. It should be emphasized here that the above result means that there are no corrections to the "polarization vector" n^ℓ_μ for the semileptonic top-quark decay. On the other hand, one can see that the corrections to α^b could be substantial as the kinematical suppression factor in the leading term $2r-1(=-0.56)$ could be canceled by the appropriate contribution from the non-standard form factor f_2^R.

Applying the strategy described above and adopting the general formula for the $t\bar{t}$ distribution $d\sigma(s_+,s_-)/d\Omega_t$ from refs.[5, 10], one obtains the following result for the double distribution of the angle and the rescaled energy of f for longitudinally polarized e^+e^- beams:

$$\frac{d^2\sigma^{(*)}}{dx_f d\cos\theta_f} = \frac{3\pi\beta\alpha_{\rm EM}^2}{2s} B_f \left[\, \Theta_0^{f(*)}(x_f) + \cos\theta_f\, \Theta_1^{f(*)}(x_f) + \cos^2\theta_f\, \Theta_2^{f(*)}(x_f) \,\right], \tag{3.3}$$

where $\alpha_{\rm EM}$ is the fine structure constant and B_f denotes the appropriate branching fraction. The energy dependence is specified by the functions $\Theta_i^{f(*)}(x_f)$, explicit forms of which for unpolarized beams were shown in ref. [11].[♯2] They are parameterized both by the production and the decay form factors.

The angular distribution for f could be easy obtained from eq.(3.3) by the integration over the energy of f:

$$\frac{d\sigma^{(*)}}{d\cos\theta_f} \equiv \int_{x_-}^{x_+} dx_f \frac{d^2\sigma^{(*)}}{dx_f d\cos\theta_f} = \frac{3\pi\beta\alpha_{\rm EM}^2}{2s} B_f \left(\Omega_0^{f(*)} + \Omega_1^{f(*)}\cos\theta_f + \Omega_2^{f(*)}\cos^2\theta_f \right), \tag{3.4}$$

where $\Omega_i^{f(*)} = \int_{x_-}^{x_+} dx \Theta_i^{f(*)}$ are shown by eq.(A.1) in the appendix and $x_\pm$ define kinematical energy range of x:

$$r(1-\beta)/(1+\beta) \le x_\ell \le 1 \quad \text{and} \quad (1-r)(1-\beta)/(1+\beta) \le x_b \le 1-r. \tag{3.5}$$

The decay vertex is entering the double distribution, eq.(3.3), through *i*) the functions $F^f(x_f)$, $G^f(x_f)$ and $H^f_{1,2}(x_f)$ defined in the appendix, and *ii*) the depolarization factor α^f. All the non-SM parts of F^f, G^f and $H^f_{1,2}$ disappear upon integration over the energy x_f both for ℓ^+ and b, as it could be seen from the explicit forms for $\Omega_i^{f(*)}$. Since $\alpha^f = 1$ for the leptonic distribution, we observe that the total dependence of the lepton distribution on non-standard structure of the top-quark decay vertex

[♯2] The functions $\Theta_i^{f(*)}(x_f)$ for polarized beams could be easily obtained from formulas for unpolarized beams replacing $D_{V,A,VA}$, $E_{V,A,VA}$, $F_{1\sim4}$, $G_{1\sim4}$ defined by eq.(A.18) with $D^{(*)}_{V,A,VA}$, $E^{(*)}_{V,A,VA}$, $F^{(*)}_{1\sim4}$, $G^{(*)}_{1\sim4}$ as in eq.(A.17) in the appendix.

drops out through the integration over the energy [11].[♯3] However, one can expect substantial modifications for the bottom-quark distribution since corrections to α^b could be large.

The fact that the angular leptonic distribution is insensitive to corrections to the $V-A$ structure of the decay vertex allows for much more clear tests of the production vertices through measurements of the distribution, since that way we can avoid a contamination from a non-standard structure of the decay vertex. As an application of the angular distribution let us consider the following CP-violating forward-backward charge asymmetry:[♯4]

$$\mathcal{A}^f_{CP}(P_{e^-},P_{e^+})=\frac{\displaystyle\int_{-c_m}^{0}d\cos\theta_f\frac{d\sigma^{+(*)}(\theta_f)}{d\cos\theta_f}-\int_{0}^{+c_m}d\cos\theta_f\frac{d\sigma^{-(*)}(\theta_f)}{d\cos\theta_f}}{\displaystyle\int_{-c_m}^{0}d\cos\theta_f\frac{d\sigma^{+(*)}(\theta_f)}{d\cos\theta_f}+\int_{0}^{+c_m}d\cos\theta_f\frac{d\sigma^{-(*)}(\theta_f)}{d\cos\theta_f}}, \tag{3.6}$$

where P_{e^-} and P_{e^+} are the polarizations of e and $\bar{e}$ beams, $d\sigma^{+/-(*)}$ is referring to f and $\bar{f}$ distributions respectively, and c_m expresses the experimental polar-angle cut. As $\theta_f \to \pi-\theta_{\bar f}$ under CP, this asymmetry is a true measure of CP violation. Since $d\sigma^{-(*)}/d\cos\theta_f$ is obtained from $d\sigma^{+(*)}/d\cos\theta_f$ by reversing the sign of $\cos\theta_f$ and $F^{(*)}_{1,4}$ terms and replacing α^f with $-\alpha^{\bar f}$ in $\Omega^{f(*)}_{0,1,2}$, the asymmetry is explicitly given by the following formula

$$\mathcal{A}^f_{CP}=N^f_A/D^f_A \tag{3.7}$$

with (in the leading order)

$$\begin{aligned}
N^f_A=&\,2c_m\alpha^f_0\Big[\,(1-c_m^2)\mathrm{Re}(F_1^{(*)})+c_m\mathrm{Re}(F_4^{(*)})\,\Big]\Big[\,1-\frac{1-\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}\,\Big]\\
&-c_m(1-\beta^2)\alpha^f_1\mathrm{Re}(f_2^R-\bar{f}_2^L)\\
&\quad\times\Big\{2(1-c_m^2)\mathrm{Re}(D_{VA}^{(0,*)})+c_mE_A^{(0,*)}\\
&\qquad-\Big[\,2(1-c_m^2)\mathrm{Re}(D_{VA}^{(0,*)})+c_m(E_V^{(0,*)}+E_A^{(0,*)})\,\Big]\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}\Big\}\\
D^f_A=&\,2c_m\Big[\,1+c_m^2\Big(1-\frac{2}{3}\beta^2\Big)\,\Big]D_V^{(0,*)}-2c_m\Big[\,(1-2\beta^2)-c_m^2\Big(1-\frac{2}{3}\beta^2\Big)\,\Big]D_A^{(0,*)}
\end{aligned}$$

[♯3]The same conclusion has also been reached through a different approach using the helicity formalism in ref.[12].

[♯4]Which is an integrated version of the asymmetry we have considered in ref.[11].

$$
\begin{aligned}
&-4c_m(1-c_m^2)\alpha_0^f(1-\beta^2)\mathrm{Re}(D_{VA}^{(0,*)}) \\
&-2c_m^2[\,\alpha_0^f(1-\beta^2)E_A^{(0,*)}+2\mathrm{Re}(E_{VA}^{(0,*)})\,] \\
&+c_m\Big\{(1-c_m^2)[\,D_V^{(0,*)}+D_A^{(0,*)}+2\alpha_0^f\mathrm{Re}(D_{VA}^{(0,*)})\,] \\
&\quad+c_m[\,\alpha_0^f(E_V^{(0,*)}+E_A^{(0,*)})+2\mathrm{Re}(E_{VA}^{(0,*)})\,]\Big\}\frac{1-\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta},
\end{aligned}
\tag{3.8}
$$

where all the coefficients are specified in the appendix, the subscript (0) indicates the SM contribution and we expressed α^f as $\alpha_0^f+\alpha_1^f\mathrm{Re}(f_2^R)$ with

$$
\begin{aligned}
&\alpha_0^f=1, &&\alpha_1^f=0 &&(\text{for } f=\ell), \\
&\alpha_0^f=\frac{2r-1}{2r+1}, &&\alpha_1^f=\frac{8\sqrt{r}(1-r)}{(1+2r)^2} &&(\text{for } f=b).
\end{aligned}
$$

As one could have anticipated, the asymmetry for $f=\ell$ is sensitive to CP violation originating exclusively from the production mechanism: It depends only on $F_{1,4}^{(*)}$ that contains contributions from the CP-violating form factors δD_γ and δD_Z while the contributing decay-vertex part consists of SM CP-conserving couplings only. For bottom quarks the effect of the modification of the decay vertex is contained in the corrections to b and $\bar{b}$ depolarization factors, $\alpha^b+\alpha^{\bar{b}}=\alpha_1^b\mathrm{Re}(f_2^R-\bar{f}_2^L)$, with SM CP-conserving contributions from the production process.[♯5]

It will be instructive to give the following remark here: The asymmetry is defined for various initial beam polarizations $P_{e^\pm}$. For $P_{e^-}\neq P_{e^+}$, the initial state seems not to be CP invariant and therefore one might expect contributions to the asymmetry originating from the CP-conserving part of the top-quark couplings. However, as it is seen from eq.(3.8), this is not the case. It turns out that even for $P_{e^-}\neq P_{e^+}$ the asymmetry is still proportional only to the CP-violating couplings embedded in $F_{1,2,3,4}$. The explanation is the following: Whatever the polarizations of the initial beams are, the electron (positron) beam consists of $e(\pm1)(\bar{e}(\pm1))$ where ±1 indicates the helicity, and only $e(\pm1)$ and $\bar{e}(\mp1)$ can interact non-trivially in the limit of $m_e=0$ since they couple to vector bosons. Therefore the interacting initial states are always CP invariant.

[♯5] One can show that $f_1^{L,R}=\pm\bar{f}_1^{L,R}$ and $f_2^{L,R}=\pm\bar{f}_2^{R,L}$ where upper (lower) signs are those for CP-conserving (-violating) contributions [13]. Therefore any CP-violating observable defined for the top-quark decay must be proportional to $f_1^{L,R}-\bar{f}_1^{L,R}$ or $f_2^{L,R}-\bar{f}_2^{R,L}$.

Now, since we have observed in ref.[11] that the differential version of the asymmetry discussed here could be substantial for higher collider energy, in order to illustrate the potential power of the asymmetry we present in tabs.1 and 2 (as a function of $\sqrt{s}$) the expected statistical significance (N_{SD}) for the asymmetry:

$$N_{SD} \equiv \frac{|\mathcal{A}^f_{CP}|}{\Delta\mathcal{A}^f_{CP}} = |\mathcal{A}^f_{CP}|\sqrt{\frac{L_{eff}\sigma_{tot}}{1-(\mathcal{A}^f_{CP})^2}}, \tag{3.9}$$

where $L_{eff} \equiv \epsilon L$ is an effective integrated luminosity for the tagging efficiency ϵ. Hereafter we adopt the integrated luminosity $L = 500$ fb^{-1} and the efficiency $\epsilon = 60\%$ both for lepton and b-quark detection.[♯6] In addition, to fit the typical

(1) $P_{e^-} = P_{e^+} = 0$

$\sqrt{s}$ (GeV)	500	700	1000	1500
$\mathcal{A}^\ell_{CP}$	$-1.2\cdot 10^{-2}$	$-2.6\cdot 10^{-2}$	$-4.0\cdot 10^{-2}$	$-5.4\cdot 10^{-2}$
N_{SD}	2.42	3.87	4.32	3.94

(2) $P_{e^-} = P_{e^+} = +0.8$

$\sqrt{s}$ (GeV)	500	700	1000	1500
$\mathcal{A}^\ell_{CP}$	$-1.4\cdot 10^{-2}$	$-2.6\cdot 10^{-2}$	$-3.6\cdot 10^{-2}$	$-4.2\cdot 10^{-2}$
N_{SD}	2.80	4.09	4.04	3.25

(3) $P_{e^-} = P_{e^+} = -0.8$

$\sqrt{s}$ (GeV)	500	700	1000	1500
$\mathcal{A}^\ell_{CP}$	$-1.2\cdot 10^{-2}$	$-2.6\cdot 10^{-2}$	$-4.1\cdot 10^{-2}$	$-5.7\cdot 10^{-2}$
N_{SD}	3.53	5.72	6.57	6.20

Table 1: The CP-violating asymmetry $\mathcal{A}^\ell_{CP}$ and the expected statistical significance N_{SD} for $\mathrm{Re}(\delta D_{\gamma,Z}) = +0.05$, and beam polarizations $P_{e^-} = P_{e^+} =$ (1) 0, (2) +0.8 and (3) −0.8 as an example.

[♯6]That low efficiency is supposed to take into account cuts necessary to suppress the background. If the b-tagging is applied then, as shown in the second paper of ref.[14], the irreducible background to top events due $W^\pm + 2\,b + 2\,j$ is negligible, provided that a vertex tagging efficiency $\epsilon_b \gtrsim .5$ can be achieved. Therefore for the b-tagging case the efficiency we have employed is definitely conservative. Since N_{SD} scales as $\sqrt{\epsilon L}$ it would be easy to estimate the statistical significance for any given luminosity and efficiency.

(1) $P_{e^-} = P_{e^+} = 0$

$\sqrt{s}$ (GeV)	500	700	1000	1500
$\mathcal{A}^b_{CP}$	$+1.2 \cdot 10^{-2}$	$+1.7 \cdot 10^{-2}$	$+2.2 \cdot 10^{-2}$	$+2.6 \cdot 10^{-2}$
N_{SD}	5.10	5.50	5.03	4.03

(2) $P_{e^-} = P_{e^+} = +0.8$

$\sqrt{s}$ (GeV)	500	700	1000	1500
$\mathcal{A}^b_{CP}$	$-9.4 \cdot 10^{-3}$	$-4.6 \cdot 10^{-3}$	$+1.4 \cdot 10^{-3}$	$+7.8 \cdot 10^{-3}$
N_{SD}	4.04	1.52	0.33	1.27

(3) $P_{e^-} = P_{e^+} = -0.8$

$\sqrt{s}$ (GeV)	500	700	1000	1500
$\mathcal{A}^b_{CP}$	$+2.6 \cdot 10^{-2}$	$+3.0 \cdot 10^{-2}$	$+3.3 \cdot 10^{-2}$	$+3.5 \cdot 10^{-2}$
N_{SD}	16.0	14.3	11.2	8.02

Table 2: The *CP*-violating asymmetry $\mathcal{A}^b_{CP}$ and the expected statistical significance N_{SD} for $\mathrm{Re}(\delta D_{\gamma,Z}) = \mathrm{Re}(f_2^R - \bar{f}_2^L) = +0.05$, and beam polarizations $P_{e^-} = P_{e^+} =$ (1) 0, (2) $+0.8$ and (3) -0.8 as an example.

detector shape [15] we impose a polar-angle cut $|\cos\theta_f| < 0.9$, i.e. $c_m = 0.9$ in eq.(3.8), both for leptons and bottom quarks. On the other hand, we will not impose any cut on the lepton/ b-quark energy since their kinematical lower bounds $E_\ell^{min} = 7.5$ GeV and $E_b^{min} = 27.5$ GeV (for $\sqrt{s} = 500$ GeV) are large enough to be detected. Perfect angular resolution will be assumed both for lepton and b-quark final states. Also ideal leptonic-energy resolution will be used.

As it is seen from the tables, the asymmetry $\mathcal{A}^f_{CP}$ turned out to be a very sensitive *CP*-violating observable; even for unpolarized beams and *CP*-violating couplings of the order of 0.05 one can expect $2.4\sigma \sim 5.5\sigma$ effect both for lepton and b-quark asymmetries once $L = 500\ \mathrm{fb}^{-1}$ is achieved.

The *CP*-violating form factors discussed here could be also generated within the SM. However, it is easy to notice that the first non-zero contribution to $\delta D_{\gamma,Z}$ would require at least two loops. For the top-quark decay process *CP* violation could appear at the one-loop level, however it is strongly suppressed by the double

GIM mechanism [16]. Therefore we can conclude that an experimental detection of CP-violating form factors considered here would be a clear indication for physics beyond the SM. In particular, non-vanishing $\mathcal{A}^l_{CP}$ in the lepton distribution will strongly indicate some new-physics in $t\bar{t}\gamma/Z$ couplings.

4. Optimal-Observable Analysis

4.1. Optimal observables

Let us briefly recall the main points of the optimal-observable (OO) technique [8]. Suppose we have a distribution

$$\frac{d\sigma}{d\phi}(\equiv \varSigma(\phi)) = \sum_i c_i f_i(\phi) \tag{4.1}$$

where $f_i(\phi)$ are known functions of the location in final-state phase space ϕ and c_i's are model-dependent coefficients. The goal would be to determine c_i's. It can be done by using appropriate weighting functions $w_i(\phi)$ such that $\int d\phi w_i(\phi)\varSigma(\phi) = c_i$. Generally, different choices for $w_i(\phi)$ are possible, but there is a unique choice so that the resultant statistical error is minimized. Such functions are given by

$$w_i(\phi) = \sum_j X_{ij} f_j(\phi)/\varSigma(\phi)\,, \tag{4.2}$$

where X_{ij} is the inverse matrix of M_{ij} which is defined as

$$M_{ij} \equiv \int d\phi \frac{f_i(\phi) f_j(\phi)}{\varSigma(\phi)}\,. \tag{4.3}$$

The statistical uncertainty of c_i-determination through $d\sigma/d\phi$ measurement becomes

$$\varDelta c_i = \sqrt{X_{ii}\,\sigma_T/N}\,, \tag{4.4}$$

where $\sigma_T \equiv \int d\phi(d\sigma/d\phi)$ and N is the total number of events.

It is clear from the definition of the matrix M_{ij}, eq.(4.3), that M_{ij} has no inverse if the functions $f_i(\phi)$ are linearly dependent, and then we cannot perform any meaningful analysis. One can see it more intuitively as follows: if $f_i(\phi) = f_j(\phi)$ the splitting between c_i and c_j would be totally arbitrary and only $c_i + c_j$ could be determined.

4.2. For application

In order to apply the OO procedure to the processes under consideration, we have to reexpress the distributions in the form shown in eq.(4.1). The angular distribution, eq.(3.4), has already an appropriate form for this purpose, where $f_i(\phi) = \cos^i\theta_f$ $(i = 0, 1, 2)$ and $\Omega_i^{f(*)}$ are the coefficients to be determined. On the other hand, the double angular and energy distribution eq.(3.3) must be modified. We reexpress the distribution in the following way, keeping only the SM contribution and terms linear in the non-standard form factors:

$$\frac{d^2\sigma^{(*)}}{dx_f d\cos\theta_f} = \frac{3\pi\beta\alpha^2}{2s} B_f\, S_f^{(*)}(x_f, \theta_f), \tag{4.5}$$

where

$$\begin{aligned} S_f^{(*)}(x_f,\theta_f) = {} & S_f^{(0,*)}(x_f,\theta_f) \\ & + \sum_{v=\gamma,Z}\Big[\,\mathrm{Re}(\delta A_v)\mathcal{F}_{Av}^{f(*)}(x_f,\theta_f) + \mathrm{Re}(\delta B_v)\mathcal{F}_{Bv}^{f(*)}(x_f,\theta_f) \\ & \qquad + \mathrm{Re}(\delta C_v)\mathcal{F}_{Cv}^{f(*)}(x_f,\theta_f) + \mathrm{Re}(\delta D_v)\mathcal{F}_{Dv}^{f(*)}(x_f,\theta_f)\,\Big] \\ & + \mathrm{Re}(f_2^R)\mathcal{F}_{2R}^{f(*)}(x_f,\theta_f). \end{aligned}$$

As it is seen from the above formula, the coefficients c_i of eq.(4.1) are just the anomalous form factors to be determined. The SM contribution reads:

$$S_f^{(0,*)}(x_f,\theta_f) = \Theta_0^{f(0,*)}(x_f) + \cos\theta_f\,\Theta_1^{f(0,*)}(x_f) + \cos^2\theta_f\,\Theta_2^{f(0,*)}(x_f) \tag{4.6}$$

with

$$\begin{aligned} \Theta_0^{f(0,*)}(x) = {} & \frac{1}{2}\Big[\,(3-\beta^2)D_V^{(0,*)} - (1-3\beta^2)D_A^{(0,*)} - 2\alpha_0^f\{\,(1-\beta^2)\mathrm{Re}(D_{VA}^{(0,*)})\,\}\,\Big]\,f^f(x) \\ & + 2\alpha_0^f\mathrm{Re}(D_{VA}^{(0,*)})\,g^f(x) + \Big[\,D_V^{(0,*)} + D_A^{(0,*)} + 2\alpha_0^f\mathrm{Re}(D_{VA}^{(0,*)})\,\Big]\,h_1^f(x) \\ & - \frac{1}{2}\Big[\,D_V^{(0,*)} + D_A^{(0,*)} + 6\alpha_0^f\,\mathrm{Re}(D_{VA}^{(0,*)})\,\Big]\,h_2^f(x), \end{aligned} \tag{4.7}$$

$$\begin{aligned} \Theta_1^{f(0,*)}(x) = {} & 2\Big[\,2\mathrm{Re}(E_{VA}^{(0,*)}) + \alpha_0^f(1-\beta^2)E_A^{(0,*)}\,\Big]f^f(x) + 2\alpha_0^f\,(E_V^{(0,*)} + E_A^{(0,*)})\,g^f(x) \\ & -2\,\Big[\,2\mathrm{Re}(E_{VA}^{(0,*)}) + \alpha_0^f(\,E_V^{(0,*)} + E_A^{(0,*)}\,)\,\Big]\,h_1^f(x), \end{aligned} \tag{4.8}$$

$$\begin{aligned} \Theta_2^{f(0,*)}(x) = {} & \Big[\,\frac{1}{2}(3-\beta^2)(D_V^{(0,*)} + D_A^{(0,*)}) + 3\alpha_0^f[\,(1-\beta^2)\mathrm{Re}(D_{VA}^{(0,*)})\,]\,\Big]\,f^f(x) \\ & + 2\alpha_0^f\mathrm{Re}(D_{VA}^{(0,*)})\,g^f(x) \\ & -3\Big[\,D_V^{(0,*)} + D_A^{(0,*)} + 2\alpha_0^f\mathrm{Re}(D_{VA}^{(0,*)})\,\Big]\,\Big[\,h_1^f(x) - \frac{1}{2}h_2^f(x)\Big]. \end{aligned} \tag{4.9}$$

Explicit forms of the functions $\mathcal{F}^{f(*)}_{\{A,B,C,D\}\{\gamma,Z\}}$ and $\mathcal{F}^{f(*)}_{2R}$ are shown in the appendix together with the functions $f^f(x)$, $g^f(x)$ and $h^f_{1,2}(x)$.

There are ten functions entering eq.(4.5): $S_f^{(0,*)}$, $\mathcal{F}^{f(*)}_{\{A,B,C,D\}\{\gamma,Z\}}$ and $\mathcal{F}^{f(*)}_{2R}$. As explained earlier, one cannot determine their coefficients separately if they are not independent. As could be found from the appendix, for the double lepton distribution, the first nine functions are linear combinations of

$$\begin{aligned}
&f^\ell(x), \quad f^\ell(x)\cos\theta, \quad f^\ell(x)\cos^2\theta, \\
&g^\ell(x), \quad g^\ell(x)\cos\theta, \quad g^\ell(x)\cos^2\theta, \\
&h^\ell_{1,2}(x)(1-3\cos^2\theta), \quad h^\ell_1(x)\cos\theta,
\end{aligned} \tag{4.10}$$

while the last one, $\mathcal{F}^{\ell(*)}_{2R}$, is a combination of $\delta\{f^\ell, g^\ell, h^\ell_{1,2}\}(x)$ and $\cos^n\theta$ ($n = 0,1,2$). Since there are ten coefficients to be measured,[♯7] it looks always possible to determine all of them. However, it turns out not to be the case in some special cases. Indeed the possibility for the determination of all the ten form factors depends crucially on the chosen beam polarization.

This can be understood considering the invariant amplitude for $e\bar{e} \to t\bar{t}$, which could be expressed in terms of eight independent parameters as

$$\begin{aligned}
\mathcal{M}(e\bar{e} \to t\bar{t}) = &\; C_{VV}\,[\,\bar{v}_e\gamma_\mu u_e \cdot \bar{u}_t\gamma^\mu v_{\bar{t}}\,] + C_{VA}\,[\,\bar{v}_e\gamma_\mu u_e \cdot \bar{u}_t\gamma_5\gamma^\mu v_t\,] \\
&+ C_{AV}\,[\,\bar{v}_e\gamma_5\gamma_\mu u_e \cdot \bar{u}_t\gamma^\mu v_t\,] + C_{AA}\,[\,\bar{v}_e\gamma_5\gamma_\mu u_e \cdot \bar{u}_t\gamma_5\gamma^\mu v_t\,] \\
&+ C_{VS}\,[\,\bar{v}_e \not{q} u_e \cdot \bar{u}_t v_t\,] + C_{VP}\,[\,\bar{v}_e \not{q} u_e \cdot \bar{u}_t\gamma_5 v_t\,] \\
&+ C_{AS}\,[\,\bar{v}_e\gamma_5 \not{q} u_e \cdot \bar{u}_t v_t\,] + C_{AP}\,[\,\bar{v}_e\gamma_5 \not{q} u_e \cdot \bar{u}_t\gamma_5 v_t\,].
\end{aligned}$$

However, if e or $\bar{e}$ is perfectly polarized, contributions from $[\bar{v}_e\gamma_\mu u_e]$ and $[\bar{v}_e\gamma_5\gamma_\mu u_e]$ are identical. For example, when e has fully left-handed polarization, u_e is replaced with $u_{eL} \equiv (1-\gamma_5)u_e/2$ and in that case they are changed as

$$\bar{v}_e\gamma_\mu u_e \;\; \to \;\; \bar{v}_e\gamma_\mu u_{eL}, \qquad \bar{v}_e\gamma_5\gamma_\mu u_e \;\; \to \;\; \bar{v}_e\gamma_\mu u_{eL}.$$

Therefore, the invariant amplitude becomes

$$\mathcal{M}(e\bar{e} \to t\bar{t})$$

[♯7] Counting the SM coefficient in front of $S_f^{(0,*)}$ which is normalized to 1.

$$= (C_{VV}+C_{AV})\,[\,\bar v_e\gamma_\mu u_{eL}\cdot \bar u_t\gamma^\mu v_{\bar t}\,] + (C_{VA}+C_{AA})\,[\,\bar v_e\gamma_\mu u_{eL}\cdot \bar u_t\gamma_5\gamma^\mu v_t\,]$$
$$+(C_{VS}+C_{AS})\,[\,\bar v_e \not{q} u_{eL}\cdot \bar u_t v_t\,] + (C_{VP}+C_{AP})\,[\,\bar v_e \not{q} u_{eL}\cdot \bar u_t\gamma_5 v_t\,],$$

and one ends with just four independent functions and therefore only four coefficients could be determined. More details could be found in the appendix below eq.(A.16). Of course, such singular configurations of polarization are not considered in our analyses.

As for b-quark distributions $\delta f^b(x) = \delta g^b(x) = \delta h_1^b(x) = \delta h_2^b(x) = 0$, instead of ten functions $\phi_i(x)$ we have in that case only nine of them given by the b-quark version eq.(4.10). Therefore, at most nine couplings could be determined. Since b-quark energy resolution is expected to be relatively poor, we will not apply OO procedure to the b-quark double distribution.

4.3. Numerical analysis

Below, we will adjust beam polarizations to perform the best measurement of the form factors. In order to gain some intuition we show in figs. 1 and 2 the functions $\mathcal{F}^{\ell(*)}_{\{A,B,C,D\}\{\gamma,Z\}}$, $\mathcal{F}^{\ell(*)}_{2R}$ plus $S^{(0,*)}_\ell$ for unpolarized beams (fig.1) and for the beam polarization $P_{e^-} = P_{e^+} = +0.5$ (fig.2). The figures illustrates how much the polarization could modify the functions and therefore influence the possibility for the determination of the form factors.

Lepton angular distribution

Since we have only three independent functions $\{1, \cos\theta, \cos^2\theta\}$, M and its inverse X are $(3,3)$ matrices. We have considered the following polarization set-ups: $P_{e^-} = P_{e^+} = 0, \pm 0.5$ and ± 1. Since $1 > |\cos\theta| > \cos^2\theta$ we observe that $X_{11} < X_{22} < X_{33}$, therefore the statistical uncertainty for $\mathit{\Omega}^{(*)}_0$ measurement, $\mathit{\Delta\Omega}^{(*)}_0$, is always the smallest one.

Once we assume the detection efficiency ϵ and the integrated luminosity L, we can compute the statistical significance of measuring the non-SM part of $\mathit{\Omega}^{(*)}_i$

$$N^{(i)}_{SD} = |\mathit{\Omega}^{(*)}_i - \mathit{\Omega}^{(0,*)}_i| / \mathit{\Delta\Omega}^{(*)}_i .$$

For the efficiency and luminosity specified earlier we obtain

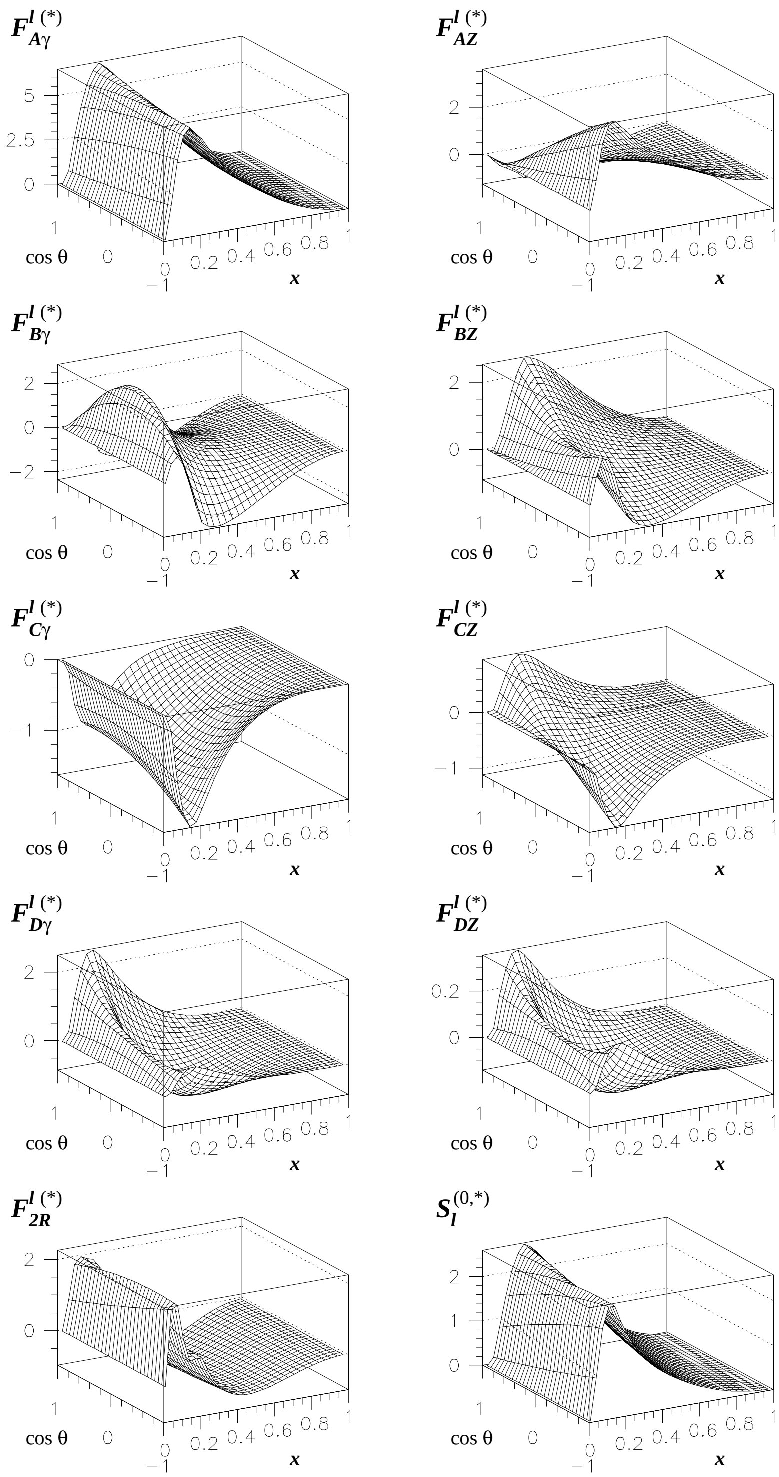

Figure 1: The shape of the coefficient functions $\mathcal{F}^{\ell(*)}_{\{A,B,C,D\}\{\gamma,Z\}}$, $\mathcal{F}^{\ell(*)}_{2R}$, and $S^{(0,*)}_{\ell}$ for unpolarized beams

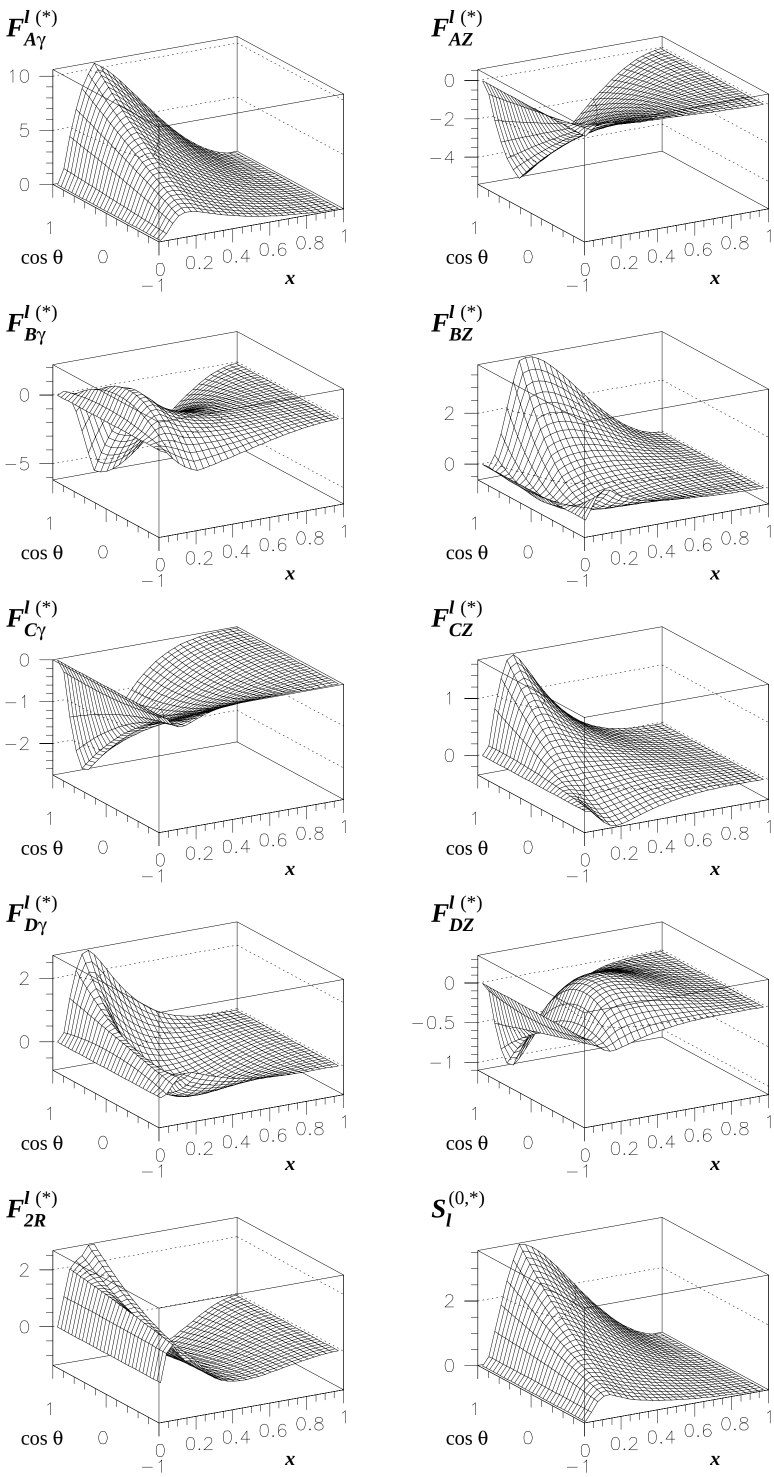

Figure 2: The shape of the coefficient functions $\mathcal{F}^{\ell(*)}_{\{A,B,C,D\}\{\gamma,Z\}}$, $\mathcal{F}^{\ell(*)}_{2R}$, and $S^{(0,*)}_{\ell}$ for $P_{e^-} = P_{e^+} = 0.5$

- $P_{e^-} = P_{e^+} = 0$

$$M_{11} = 2.06, \quad M_{22} = 0.55, \quad M_{33} = 0.27$$
$$X_{11} = 1.09, \quad X_{22} = 1.83, \quad X_{33} = 8.40$$
$$N_{SD}^{(0)} = 16.1, \quad N_{SD}^{(1)} = 3.5, \quad N_{SD}^{(2)} = 1.9$$

- $P_{e^-} = P_{e^+} = +0.5$

$$M_{11} = 3.06, \quad M_{22} = 0.95, \quad M_{33} = 0.49$$
$$X_{11} = 0.82, \quad X_{22} = 1.66, \quad X_{33} = 6.25$$
$$N_{SD}^{(0)} = 7.5, \quad N_{SD}^{(1)} = 2.7, \quad N_{SD}^{(2)} = 1.3$$

- $P_{e^-} = P_{e^+} = +1$

$$M_{11} = 3.20, \quad M_{22} = 1.25, \quad M_{33} = 0.72$$
$$X_{11} = 1.00, \quad X_{22} = 2.39, \quad X_{33} = 7.00$$
$$N_{SD}^{(0)} = 5.2, \quad N_{SD}^{(1)} = 3.0, \quad N_{SD}^{(2)} = 1.3$$

- $P_{e^-} = P_{e^+} = -0.5$

$$M_{11} = 1.27, \quad M_{22} = 0.35, \quad M_{33} = 0.17$$
$$X_{11} = 1.81, \quad X_{22} = 2.91, \quad X_{33} = 13.4$$
$$N_{SD}^{(0)} = 26.2, \quad N_{SD}^{(1)} = 4.9, \quad N_{SD}^{(2)} = 3.0$$

- $P_{e^-} = P_{e^+} = -1$

$$M_{11} = 0.76, \quad M_{22} = 0.21, \quad M_{33} = 0.11$$
$$X_{11} = 3.06, \quad X_{22} = 4.90, \quad X_{33} = 22.3$$
$$N_{SD}^{(0)} = 35.5, \quad N_{SD}^{(1)} = 6.4, \quad N_{SD}^{(2)} = 4.0, \tag{4.11}$$

where we put all the non-SM parameters $\mathrm{Re}(\delta\{A,B,C,D\}_{\gamma,Z})$ and $\mathrm{Re}(f_2^R)$ to be $+0.05$ as an example.

As one can see, the precision is better for negative beam polarization, partly because of larger number of events. However we cannot conclude that using negatively-polarized beams is always more effective for new-physics search, since $N_{SD}^{(i)}$ strongly depends on the non-SM parameters used in the computations. In fact, positively-polarized beams give smaller X_{ii} and this is independent of the choice of non-SM parameters. Therefore polarization of the initial beams should be carefully adjusted for each tested model in actual experimental analysis.

b-quark angular distribution

We can compute M, X and $N_{SD}^{(i)}$ in the same way as for the lepton distribution:

• $P_{e^-} = P_{e^+} = 0$

$$M_{11} = 2.23, \quad M_{22} = 0.63, \quad M_{33} = 0.31$$

$$X_{11} = 1.04, \quad X_{22} = 1.85, \quad X_{33} = 7.98$$

$$N_{SD}^{(0)} = 37.3, \quad N_{SD}^{(1)} = 17.8, \quad N_{SD}^{(2)} = 1.9$$

• $P_{e^-} = P_{e^+} = +0.5$

$$M_{11} = 2.38, \quad M_{22} = 0.65, \quad M_{33} = 0.32$$

$$X_{11} = 0.96, \quad X_{22} = 1.61, \quad X_{33} = 7.29$$

$$N_{SD}^{(0)} = 15.5, \quad N_{SD}^{(1)} = 7.6, \quad N_{SD}^{(2)} = 1.8$$

• $P_{e^-} = P_{e^+} = +1$

$$M_{11} = 1.63, \quad M_{22} = 0.45, \quad M_{33} = 0.22$$

$$X_{11} = 1.39, \quad X_{22} = 2.29, \quad X_{33} = 10.5$$

$$N_{SD}^{(0)} = 9.7, \quad N_{SD}^{(1)} = 4.8, \quad N_{SD}^{(2)} = 2.2$$

• $P_{e^-} = P_{e^+} = -0.5$

$$M_{11} = 1.45, \quad M_{22} = 0.42, \quad M_{33} = 0.21$$

$$X_{11} = 1.63, \quad X_{22} = 3.01, \quad X_{33} = 12.5$$

$$N_{SD}^{(0)} = 62.6, \quad N_{SD}^{(1)} = 29.3, \quad N_{SD}^{(2)} = 2.4$$

• $P_{e^-} = P_{e^+} = -1$

$$M_{11} = 0.87, \quad M_{22} = 0.25, \quad M_{33} = 0.13$$

$$
\begin{aligned}
&X_{11} = 2.74, \quad X_{22} = 5.10, \quad X_{33} = 21.0 \\
&N_{SD}^{(0)} = 85.1, \quad N_{SD}^{(1)} = 39.7, \quad N_{SD}^{(2)} = 3.1,
\end{aligned} \tag{4.12}
$$

for $\mathrm{Re}(\delta\{A,B,C,D\}_{\gamma,Z}) = \mathrm{Re}(f_2^R) = +0.05$. Negatively-polarized beams give better precision again, but the same remark as to the lepton angular distribution should be kept in mind also here.

The above results prove that the optimal observables utilizing the angular distributions should be very efficient seeking for the non-SM parts of $\Omega_i^{f(*)}$. However, since they are combinations of the form factors, we can only constrain them. Of course, it would be exciting if we found any signal of non-standard physics, however our final goal is to determine each form factor separately. That is why we proceed to the next analysis using the double angular and energy distributions.

Lepton angular and energy distribution

Because of high precision of direction and energy determination of leptons we adopted the double energy and angular distributions, eq.(4.5), also for OO analysis. As discussed earlier, in principle all nine form factors could be determined with the expected statistical uncertainties Δc_i for $c_i = \mathrm{Re}(\delta\{A,B,C,D\}_{\gamma,Z})$ and $\mathrm{Re}(f_2^R)$. The beam polarizations P_{e^-} and P_{e^+} were adjusted to minimize the statistical error for determination of each form factor. We found that positive polarizations lead to a smaller Δc_i for eight form factors in the production vertices. Unfortunately, however, the optimal polarizations for each form-factor measurement is different. Below we present the smallest statistical uncertainties and the corresponding beam polarizations for each parameter:

$$
\Delta[\,\mathrm{Re}(\delta A_\gamma)\,] = 0.16 \qquad \text{for } P_{e^-} = 0.7 \text{ and } P_{e^+} = 0.7,
$$
$$
\begin{pmatrix} \delta A_Z\!: 0.13,\ \delta B_\gamma\!: 0.25,\ \delta B_Z\!: 0.49,\ \delta C_\gamma\!: 2.47 \\ \delta C_Z\!: 4.69,\ \delta D_\gamma\!: 27.2,\ \delta D_Z\!: 53.2,\ f_2^R\!: 0.02 \end{pmatrix}
$$
$$
\Delta[\,\mathrm{Re}(\delta A_Z)\,] = 0.07 \qquad \text{for } P_{e^-} = 0.5 \text{ and } P_{e^+} = 0.4,
$$
$$
\begin{pmatrix} \delta A_\gamma\!: 0.23,\ \delta B_\gamma\!: 0.11,\ \delta B_Z\!: 0.27,\ \delta C_\gamma\!: 0.70 \\ \delta C_Z\!: 1.76,\ \delta D_\gamma\!: 7.09,\ \delta D_Z\!: 20.6,\ f_2^R\!: 0.02 \end{pmatrix}
$$
$$
\Delta[\,\mathrm{Re}(\delta B_\gamma)\,] = 0.09 \qquad \text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.2,
$$
$$
\begin{pmatrix} \delta A_\gamma\!: 0.43,\ \delta A_Z\!: 0.11,\ \delta B_Z\!: 0.36,\ \delta C_\gamma\!: 0.21 \\ \delta C_Z\!: 1.17,\ \delta D_\gamma\!: 0.95,\ \delta D_Z\!: 14.6,\ f_2^R\!: 0.03 \end{pmatrix}
$$

$$\mathit{\Delta}[\,\mathrm{Re}(\delta B_Z)\,] = 0.27 \qquad \text{for } P_{e^-} = 0.4 \text{ and } P_{e^+} = 0.4,$$
$$\begin{pmatrix} \delta A_\gamma\!: 0.25,\ \delta A_Z\!: 0.07,\ \delta B_\gamma\!: 0.10,\ \delta C_\gamma\!: 0.56 \\ \delta C_Z\!: 1.56,\ \delta D_\gamma\!: 5.43,\ \delta D_Z\!: 18.5,\ f_2^R\!: 0.02 \end{pmatrix}$$
$$\mathit{\Delta}[\,\mathrm{Re}(\delta C_\gamma)\,] = 0.11 \qquad \text{for } P_{e^-} = 0.1 \text{ and } P_{e^+} = 0.0,$$
$$\begin{pmatrix} \delta A_\gamma\!: 0.82,\ \delta A_Z\!: 0.22,\ \delta B_\gamma\!: 0.10,\ \delta B_Z\!: 0.65 \\ \delta C_Z\!: 1.11,\ \delta D_\gamma\!: 1.76,\ \delta D_Z\!: 14.6,\ f_2^R\!: 0.03 \end{pmatrix}$$
$$\mathit{\Delta}[\,\mathrm{Re}(\delta C_Z)\,] = 1.11 \qquad \text{for } P_{e^-} = 0.1 \text{ and } P_{e^+} = 0.0,$$
$$\begin{pmatrix} \delta A_\gamma\!: 0.82,\ \delta A_Z\!: 0.22,\ \delta B_\gamma\!: 0.10,\ \delta B_Z\!: 0.65 \\ \delta C_\gamma\!: 0.11,\ \delta D_\gamma\!: 1.76,\ \delta D_Z\!: 14.6,\ f_2^R\!: 0.03 \end{pmatrix}$$
$$\mathit{\Delta}[\,\mathrm{Re}(\delta D_\gamma)\,] = 0.08 \qquad \text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.1,$$
$$\begin{pmatrix} \delta A_\gamma\!: 0.52,\ \delta A_Z\!: 0.13,\ \delta B_\gamma\!: 0.09,\ \delta B_Z\!: 0.42 \\ \delta C_\gamma\!: 0.15,\ \delta C_Z\!: 1.13,\ \delta D_Z\!: 14.4,\ f_2^R\!: 0.03 \end{pmatrix}$$
$$\mathit{\Delta}[\,\mathrm{Re}(\delta D_Z)\,] = 14.4 \qquad \text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.1,$$
$$\begin{pmatrix} \delta A_\gamma\!: 0.64,\ \delta A_Z\!: 0.17,\ \delta B_\gamma\!: 0.09,\ \delta B_Z\!: 0.51 \\ \delta C_\gamma\!: 0.12,\ \delta C_Z\!: 1.12,\ \delta D_\gamma\!: 0.84,\ f_2^R\!: 0.03 \end{pmatrix} \tag{4.13}$$

where we also showed the expected precision of the other parameter measurements for the same beam polarizations. For instance, we can expect $\mathit{\Delta}[\,\mathrm{Re}(\delta A_\gamma)\,] = 0.16$ for $P_{e^-} = P_{e^+} = 0.7$ while the expected precision of $\mathrm{Re}(\delta A_Z)$, $\mathrm{Re}(\delta B_\gamma)$, $\cdots$ for the same polarizations are 0.13, 0.25, $\cdots$, respectively. This result is independent of the choice of the non-SM parameters in contrast to the preceding results.

As it is seen the precision of $\delta\{C,D\}_Z$ measurement would be very poor even for the optimal polarization. This is mainly a consequence of the size of $\mathcal{F}^{\ell(*)}_{\{C,D\}Z}$, which is illustrated in figs.1 and 2: These two functions are very small in a large area. More quantitatively, the size of the elements of M matrix, M_{ij}, is $O(1)$ for $i,j \neq 7,9$, while the size of $M_{i7}(=M_{7i})$ and $M_{i9}(=M_{9i})$ is at most $O(10^{-2})$. In addition, determination of δD_γ would be practically difficult, as well, since its error varies rapidly with the polarization. For example, $\mathit{\Delta}[\,\mathrm{Re}(\delta D_\gamma)\,]$ becomes 0.86 for $P_{e^-} = 0.1/P_{e^+} = 0.1$ and 0.99 for $P_{e^-} = 0.3/P_{e^+} = 0.1$. The source of that sensitivity is hidden in the neutral-current structure with $\sin^2\theta_W \simeq 0.23$. Indeed, the optimal polarization becomes $P_{e^-} = 0.1$ instead of 0.2 ($\mathit{\Delta}[\,\mathrm{Re}(\delta D_\gamma)\,] = 0.09$) for $\sin^2\theta_W = 0.25$. On the other hand, a good determination (almost independently of the polarization) could be expected for f_2^R. Indeed, the best precision is

$$\mathit{\Delta}[\,\mathrm{Re}(f_2^R)\,] = 0.01 \qquad \text{for } P_{e^-} = -0.8 \text{ and } P_{e^+} = -0.8 \tag{4.14}$$

whereas even for the unpolarized beams we obtain $\Delta[\,\mathrm{Re}(f_2^R)\,] = 0.03$.

At the time a linear collider will be operating, data from Tevatron Run II and LHC will also provide independent constraints on top-quark couplings. Below we provide an example of a combined analysis assuming δA_v, δB_v and f_2^R are known and OO are used to determine δC_v and δD_v only (here we put $\delta A_v = \delta B_v = f_2^R = 0$ for simplicity). The results are as follows:

$$\begin{aligned}
&\Delta[\,\mathrm{Re}(\delta C_\gamma)\,] = 0.04 &&\text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.2\\
&\Delta[\,\mathrm{Re}(\delta C_Z)\,] = 0.23 &&\text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.1\\
&\Delta[\,\mathrm{Re}(\delta D_\gamma)\,] = 0.03 &&\text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.1\\
&\Delta[\,\mathrm{Re}(\delta D_Z)\,] = 2.97 &&\text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.1
\end{aligned} \tag{4.15}$$

The error for δD_Z became much smaller but still too large for practical use. However, as we have seen in sec.3, the CP-sensitive asymmetry $\mathcal{A}^f_{CP}$ would provide much stronger constraints on $\delta D_{\gamma,Z}$.

5. Summary and Conclusions

We have presented here the angular and energy distributions for $\overset{(-)}{f}$ in the process $e^+e^- \to t\bar{t} \to \overset{(-)}{f} \cdots$, where $f = \ell$ or b quark in the form suitable for an application of the optimal observables (OO). The most general (CP-violating *and* CP-conserving) couplings for $\gamma t\bar{t}$, $Zt\bar{t}$ and Wtb have been assumed. All fermion masses except m_t have been neglected and we have kept only terms linear in anomalous couplings. We have assumed the tagging efficiency at the level of 60% both for lepton and b quark detection, the range of the polar angle restricted by $|\cos\theta_f| < 0.9$ and the integrated luminosity $L = 500\ \mathrm{fb}^{-1}$.

CP-violating charge forward-backward asymmetry $\mathcal{A}^f_{CP}$ has been introduced as an efficient way for testing CP-violation in top-quark couplings. Since the angular distribution for leptons is insensitive to variations of the standard V$-$A structure of the Wtb coupling, the asymmetry could be utilized for a pure test of CP-violation in the top-quark production process. The expected statistical significance N_{SD} for the measurement of the asymmetry has been calculated. We have found that it should be possible to detect $\mathcal{A}^f_{CP}$ at 5.5σ (4.3σ) level for bottom quarks (leptons) for unpolarized beams, assuming CP-violating couplings of the order of 0.05. Having both beams polarized at 80% the signal for bottom quarks (leptons) could reach even 16σ (6.6σ).

Next, the OO procedure has been applied to the angular distributions. In the case of the lepton angular distribution, the expected statistical significance for signals of non-standard physics varies between 1.3σ and 35.5σ assuming non-standard form factors of the order of 0.05. It turned out that in the case of the bottom-quark angular distribution the statistical significance of the signal is in general higher than for leptons because of larger event rate and varies between 1.8σ and 85.1σ for the same non-standard form factors.

When deriving the above results we have fixed all the non-SM parameters to be $+0.05$ as a reasonable example for the strength of beyond-the-SM physics. However, final results for statistical significances considered here depend on the size of the non-standard parameters. The most convenient beam polarizations for a measurement of the asymmetry $\mathcal{A}^f_{CP}$ and for testing the angular distributions varies with the non-standard parameters, as well. Therefore one should stress that the beam polarizations should be carefully adjusted for each model to be tested in actual experimental analysis. However, in any case, the above results show that a measurement of $\mathcal{A}^f_{CP}$ and OO analysis of the angular distributions are both very efficient for new-physics search.

Then we have analyzed the angular and energy distribution of the lepton toward separate determinations of the anomalous form factors. In order to reach the highest precision we have been adjusting beam polarizations to minimize errors for each form factor. We have found that at $\sqrt{s} = 500$ GeV with the integrated luminosity $L = 500\ \mathrm{fb}^{-1}$ the best determined coupling would be the axial coupling of the Z boson with the error $\mathit{\Delta}[\,\mathrm{Re}(\delta A_Z)\,] = 0.07$ while the lowest precision is expected for $\mathrm{Re}(\delta D_Z)$ with $\mathit{\Delta}[\,\mathrm{Re}(\delta D_Z)\,] = 14.4$. This result is independent of the choice of the non-SM parameters in contrast to the above two types of analyses.

Concluding, we have observed that the angular distributions and the angular and energy distributions of top-quark decay products both provide very efficient tools for studying top-quark couplings to gauge bosons at linear colliders.

ACKNOWLEDGMENTS

We would like to thank K. Fujii for useful discussions concerning details of lepton and bottom-quark detection and J. Pliszka for his remarks on the statistical analysis. One of us (Z.H.) is grateful to Y. Sumino, T. Nagano, T. Takahashi and K. Ikematsu for stimulating discussions. This work is supported in part by the State Committee for Scientific Research (Poland) under grant 2 P03B 014 14 and by Maria Skłodowska-Curie Joint Fund II (Poland-USA) under grant MEN/NSF-96-252.

Appendix

Integrals of $\Theta_i^{f(*)}(x)$ denoted in the main text by $\Omega_i^{f(*)}$ in the angular distribution eq.(3.4) are the following:

$$\begin{aligned}
\Omega_0^{f(*)} &= D_V^{(*)} - (1-2\beta^2)D_A^{(*)} - 2\,\mathrm{Re}(G_1^{(*)}) \\
&\quad - \alpha^f[\,2(1-\beta^2)\mathrm{Re}(D_{VA}^{(*)}) - \mathrm{Re}(F_1^{(*)}) + (3-2\beta^2)\mathrm{Re}(G_3^{(*)})\,] \\
&\quad + \Big[\,D_V^{(*)} + D_A^{(*)} + 2\,\mathrm{Re}(G_1^{(*)}) \\
&\qquad\quad + \alpha^f\mathrm{Re}(2D_{VA}^{(*)} - F_1^{(*)} + 3G_3^{(*)})\,\Big]\frac{1-\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}, \\
\Omega_1^{f(*)} &= 4\,\mathrm{Re}(E_{VA}^{(*)}) + 2\alpha^f[\,(1-\beta^2)E_A^{(*)} - \mathrm{Re}(F_4^{(*)} - G_2^{(*)})\,] \\
&\quad - \{2\mathrm{Re}(E_{VA}^{(*)}) + \alpha^f[\,E_V^{(*)} + E_A^{(*)} - \mathrm{Re}(F_4^{(*)} - G_2^{(*)})\,]\,\}\frac{1-\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta}, \\
\Omega_2^{f(*)} &= (3-2\beta^2)[\,D_V^{(*)} + D_A^{(*)} + 2\mathrm{Re}(G_1^{(*)})\,] \\
&\quad + 3\alpha^f[\,2(1-\beta^2)\mathrm{Re}(D_{VA}^{(*)}) - \mathrm{Re}(F_1^{(*)}) + (3-2\beta^2)\mathrm{Re}(G_3^{(*)})\,] \\
&\quad - 3\Big[\,D_V^{(*)} + D_A^{(*)} + 2\,\mathrm{Re}(G_1^{(*)}) \\
&\qquad\quad + \alpha^f\mathrm{Re}(2D_{VA}^{(*)} - F_1^{(*)} + 3G_3^{(*)})\,\Big]\frac{1-\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}.
\end{aligned} \tag{A.1}$$

Next we present explicit formulas of the coefficient functions for the nine anomalous form factors in eq.(4.5) $\mathcal{F}^{f(*)}_{\{A,B,C,D\}\{\gamma,Z\}}(x,\theta)$ and $\mathcal{F}^{f(*)}_{2R}(x,\theta)$ $(f=\ell/b)$:

$$
\begin{aligned}
&\mathcal{F}^{f(*)}_{Av}(x,\theta)\\
&=\Big[\,\frac{1}{2}(3-\beta^2)C(D_V\!:\!A_v)f^f(x)+2\alpha_0^fC(D_{VA}\!:\!A_v)g^f(x)\,\Big](1+\cos^2\theta)\\
&-\Big[\,\alpha_0^f(1-\beta^2)C(D_{VA}\!:\!A_v)f^f(x)\\
&\qquad-\frac{1}{2}\{C(D_V\!:\!A_v)+2\alpha_0^fC(D_{VA}\!:\!A_v)\}\{2h_1^f(x)-h_2^f(x)\}\,\Big](1-3\cos^2\theta)\\
&+2\Big[\,\alpha_0^fC(E_V\!:\!A_v)\{g^f(x)-h_1^f(x)\}+2C(E_{VA}\!:\!A_v)\{f^f(x)-h_1^f(x)\}\,\Big]\cos\theta,
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
&\mathcal{F}^{f(*)}_{Bv}(x,\theta)\\
&=\frac{1}{2}\beta^2C(D_A\!:\!B_v)f^f(x)(3-\cos^2\theta)+2\alpha_0^fC(D_{VA}\!:\!B_v)g^f(x)(1+\cos^2\theta)\\
&-\frac{1}{2}\Big[\,\{C(D_A\!:\!B_v)+2\alpha_0^f(1-\beta^2)C(D_{VA}\!:\!A_v)\}f^f(x)\\
&\qquad-\{C(D_A\!:\!B_v)+2\alpha_0^fC(D_{VA}\!:\!B_v)\}\{2h_1^f(x)-h_2^f(x)\}\,\Big](1-3\cos^2\theta)\\
&+2\Big[\,\{\alpha_0^f(1-\beta^2)C(E_A\!:\!B_v)+2C(E_{VA}\!:\!B_v)\}f^f(x)+\alpha_0^fC(E_A\!:\!B_v)g^f(x)\\
&\qquad-\{\alpha_0^fC(E_A\!:\!B_v)+2C(E_{VA}\!:\!B_v)\}h_1^f(x)\,\Big]\cos\theta,
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
&\mathcal{F}^{f(*)}_{Cv}(x,\theta)\\
&=-\beta^2C(G_1\!:\!C_v)f^f(x)(1+\cos^2\theta)\\
&+2\alpha_0^fC(G_2\!:\!C_v)\Big[\,f^f(x)+g^f(x)-h_1^f(x)\,\Big]\cos\theta\\
&-\Big[\,\{C(G_1\!:\!C_v)+\alpha_0^f(2-\beta^2)C(G_3\!:\!C_v)\}f^f(x)+\alpha_0^fC(G_3\!:\!C_v)g^f(x)\\
&\qquad-\{2C(G_1\!:\!C_v)+3\alpha_0^fC(G_3\!:\!C_v)\}h_1^f(x)\\
&\qquad+\{C(G_1\!:\!C_v)+\alpha_0^fC(G_3\!:\!C_v)\}h_2^f(x)\,\Big](1-3\cos^2\theta)
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
&\mathcal{F}^{f(*)}_{Dv}(x,\theta)\\
&=\alpha_0^fC(F_1\!:\!D_v)\Big[\,f^f(x)-h_1^f(x)\,\Big](1-3\cos^2\theta)-\alpha_0^fC(F_1\!:\!D_v)g^f(x)(1+\cos^2\theta)\\
&\qquad-2\alpha_0^fC(F_4\!:\!D_v)\Big[\,f^f(x)+g^f(x)-h_1^f(x)\,\Big]\cos\theta,
\end{aligned}
\tag{A.5}
$$

while $\mathcal{F}^{\ell(*)}_{2R}(x,\theta)$ takes different forms for $f=\ell$ and $f=b$ as

$$
\begin{aligned}
&\mathcal{F}_{2R}^{\ell(*)}(x,\theta) \\
&= \frac{1}{2}\Big[\,(3-\beta^2)D_V^{(0,*)} - (1-3\beta^2)D_A^{(0,*)} - 2(1-\beta^2)\mathrm{Re}(D_{VA}^{(0,*)})\,\Big]\,\delta f^{\ell}(x) \\
&+2\mathrm{Re}(D_{VA}^{(0,*)})\;\delta g^{\ell}(x)(1+\cos^2\theta) \\
&+\frac{1}{2}\Big[\,D_V^{(0,*)} + D_A^{(0,*)} + 2\mathrm{Re}(D_{VA}^{(0,*)})\,\Big]\Big[\,2\delta h_1^{\ell}(x) - \delta h_2^{\ell}(x)\,\Big](1-3\cos^2\theta) \\
&+2\Big[\,(1-\beta^2)E_A^{(0,*)} + 2\mathrm{Re}(E_{VA}^{(0,*)})\,\Big]\delta f^{\ell}(x)\cos\theta \\
&+2\;(\,E_V^{(0,*)} + E_A^{(0,*)}\,)\;\delta g^{\ell}(x)\cos\theta \\
&-2\;\Big[\,E_V^{(0,*)} + E_A^{(0,*)} + 2\mathrm{Re}(E_{VA}^{(0,*)})\,\Big]\;\delta h_1^{\ell}(x)\cos\theta \\
&+\frac{1}{2}\Big[\,(3-\beta^2)(D_V^{(0,*)} + D_A^{(0,*)}) + 6(1-\beta^2)\mathrm{Re}(D_{VA}^{(0,*)})\,\Big]\,\delta f^{\ell}(x)\cos^2\theta, \qquad \text{(A.6)}
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathcal{F}_{2R}^{b(*)}(x,\theta) \\
&= \alpha_1^b\,\Big\{\mathrm{Re}(D_{VA}^{(0,*)})\Big[-\Big\{(1-\beta^2)f^b(x) - 2h_1^b(x) + h_2^b(x)\Big\}(1-3\cos^2\theta) \\
&\qquad\qquad + 2g^b(x)(1+\cos^2\theta)\,\Big] \\
&+2\Big[\,(1-\beta^2)E_A^{(0,*)}f^b(x) + (E_V^{(0,*)} + E_A^{(0,*)})\Big\{g^b(x) - h_1^b(x)\Big\}\,\Big]\cos\theta\Big\}, \quad \text{(A.7)}
\end{aligned}
$$

where the functions $f^f(x)$, $g^f(x)$, $h_{1,2}^f(x)$, $\delta f^f(x)$, $\delta g^f(x)$ and $\delta h_{1,2}^f(x)$ are defined as

$$
\begin{aligned}
F^f(x) &= f^f(x) + \mathrm{Re}(f_2^R)\delta f^f(x), \\
G^f(x) &= g^f(x) + \mathrm{Re}(f_2^R)\delta g^f(x), \\
H_{1,2}^f(x) &= h_{1,2}^f(x) + \mathrm{Re}(f_2^R)\delta h_{1,2}^f(x), \qquad \text{(A.8)}
\end{aligned}
$$

with $F^f(x)$, $G^f(x)$ and $H_{1,2}^f(x)$ being given as follows [11]

$$
\begin{aligned}
&F^f(x) \equiv \frac{1}{B_f}\int d\omega\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dxd\omega}, \quad G^f(x) \equiv \frac{1}{B_f}\int d\omega\Big[\,1 - x\frac{1+\beta}{1-\omega}\Big]\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dxd\omega}, \\
&H_1^f(x) \equiv \frac{1}{B_f}\frac{1-\beta}{x}\int d\omega(1-\omega)\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dxd\omega}, \\
&H_2^f(x) \equiv \frac{1}{B_f}\Big(\frac{1-\beta}{x}\Big)^2\int d\omega(1-\omega)^2\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dxd\omega}, \qquad \text{(A.9)}
\end{aligned}
$$

and ω is defined as $\omega \equiv (p_t - p_f)^2/m_t^2$.

After performing the above integrations using

$$\frac{1}{\Gamma_t}\frac{d^2\Gamma_f}{dxd\omega} = \begin{cases} \dfrac{1+\beta}{\beta}\dfrac{3B_\ell}{W}\omega\left[\,1+2\mathrm{Re}(f_2^R)\sqrt{r}\left(\dfrac{1}{1-\omega}-\dfrac{3}{1+2r}\right)\right] & \text{for } \ f=\ell^+, \\ \\ \dfrac{1+\beta}{2\beta(1-r)}\delta(\omega-r) & \text{for } \ f=b. \end{cases}$$

one obtains the following explicit forms of $f^f(x)$, $g^f(x)$, $h^f_{1,2}(x)$, $\delta f^f(x)$, $\delta g^f(x)$ and $\delta h^f_{1,2}(x)$ for leptonic and bottom-quark final states:

- For $f=\ell$

$$\begin{aligned} f^\ell(x) &= \frac{3(1+\beta)}{2\beta W}[\,\omega^2\,]^{\omega_+}_{\omega_-}\ \left(\equiv \frac{3(1+\beta)}{2\beta W}(\omega_+^2-\omega_-^2)\right), \\ g^\ell(x) &= f^\ell(x)+\frac{3(1+\beta)^2}{\beta W}x\Big[\,\omega+\ln|1-\omega|\,\Big]^{\omega_+}_{\omega_-}, \\ h_1^\ell(x) &= \frac{1-\beta^2}{2\beta W}\frac{1}{x}\Big[\,\omega^2(3-2\omega)\Big]^{\omega_+}_{\omega_-}, \\ h_2^\ell(x) &= \frac{1}{4\beta W}(1+\beta)(1-\beta)^2\frac{1}{x^2}\Big[\,\omega^2(6-8\omega+3\omega^2)\,\Big]^{\omega_+}_{\omega_-}, \end{aligned}$$

$$\begin{aligned} \delta f^\ell(x) &= -\frac{3(1+\beta)}{\beta W}\sqrt{r}\Big[\,2\omega+2\ln|1-\omega|+\frac{3\omega^2}{1+2r}\,\Big]^{\omega_+}_{\omega_-}, \\ \delta g^\ell(x) &= \delta f^\ell(x)-\frac{6(1+\beta)^2}{\beta W}x\sqrt{r}\Big[\,\ln|1-\omega| \\ &\qquad\qquad +\frac{1}{1-\omega}+\frac{3}{1+2r}(\omega+\ln|1-\omega|)\,\Big]^{\omega_+}_{\omega_-}, \\ \delta h_1^\ell(x) &= \frac{3(1-\beta^2)}{\beta W}\frac{\sqrt{r}}{x}\Big[\,\omega^2\Big(1-\frac{3-2\omega}{1+2r}\Big)\,\Big]^{\omega_+}_{\omega_-}, \\ \delta h_2^\ell(x) &= \frac{1}{2\beta W}(1+\beta)(1-\beta)^2 \\ &\qquad \times\frac{\sqrt{r}}{x^2}\Big[\,\omega^2+\Big\{2(3-2\omega)-\frac{3}{1+2r}(6-8\omega+3\omega^2)\Big\}\,\Big]^{\omega_+}_{\omega_-}, \end{aligned} \tag{A.10}$$

where $\omega_\pm$ are given as follows:

For $r\geq B$ $\ (r\equiv M_W^2/m_t^2$ and $B\equiv(1-\beta)/(1+\beta))$

$$\begin{aligned} &\omega_+=1-r, \quad \omega_-=1-x/B && \text{for} \quad Br\leq x<B \\ &\omega_+=1-r, \quad \omega_-=0 && \text{for} \quad B\leq x<r \\ &\omega_+=1-x, \quad \omega_-=0 && \text{for} \quad r\leq x\leq 1 \end{aligned} \tag{A.11}$$

For $r < B$

$$\begin{aligned} &\omega_+ = 1 - r, \quad \omega_- = 1 - x/B && \text{for} \quad Br \le x < r \\ &\omega_+ = 1 - x, \quad \omega_- = 1 - x/B && \text{for} \quad r \le x < B \\ &\omega_+ = 1 - x, \quad \omega_- = 0 && \text{for} \quad B \le x \le 1 \end{aligned} \tag{A.12}$$

- For $f = b$

$$\begin{aligned} f^b(x) &= \frac{1+\beta}{2\beta(1-r)} \; (= \text{constant}), \\ g^b(x) &= \Big(1 - \frac{1+\beta}{1-r}x\Big)\frac{1+\beta}{2\beta(1-r)}, \\ h_1^b(x) &= \frac{1-\beta^2}{2\beta x}, \\ h_2^b(x) &= \frac{(1-r)(1+\beta)(1-\beta)^2}{2\beta x^2}, \\ \delta f^b(x) &= \delta g^b(x) = \delta h_1^b(x) = \delta h_2^b(x) = 0, \end{aligned} \tag{A.13}$$

where x is bounded as

$$B(1-r) \le x \le 1-r.$$

The coefficients $C(X:Y)$ employed in the definition of the coefficient functions have been introduced through the following formulas:

$$\begin{aligned} D_V^{(*)} &= D_V^{(0,*)} + \sum_{v=\gamma,Z} C(D_V\!:\!A_v)\mathrm{Re}(\delta A_v), \\ D_A^{(*)} &= D_A^{(0,*)} + \sum_{v=\gamma,Z} C(D_A\!:\!B_v)\mathrm{Re}(\delta B_v), \\ \mathrm{Re}(D_{VA}^{(*)}) &= \mathrm{Re}(D_{VA}^{(0,*)}) \\ &\qquad + \sum_{v=\gamma,Z}\Big[\, C(D_{VA}\!:\!A_v)\mathrm{Re}(\delta A_v) + C(D_{VA}\!:\!B_v)\mathrm{Re}(\delta B_v)\,\Big], \end{aligned} \tag{A.14}$$

and in the analogous manner for $E_{V,A,VA}$, $F_{1\sim4}$ and $G_{1\sim4}$. $D_{V,A,VA}^{(0,*)}$, $E_{V,A,VA}^{(0,*)}$, $F_{1\sim4}^{(0,*)}$, $G_{1\sim4}^{(0,*)}$ could be obtained from eq.(A.17) below as a SM approximation of $D_{V,A,VA}^{(*)}$, $E_{V,A,VA}^{(*)}$, $F_{1\sim4}^{(*)}$, $G_{1\sim4}^{(*)}$. Explicit forms of the independent coefficients are given as

$$\begin{aligned} C(D_V\!:\!A_\gamma) &= 2C[\,\mathcal{P}_\otimes A_\gamma - (\mathcal{P}_\oplus + v_e\mathcal{P}_\otimes)d'A_Z\,], \\ C(E_V\!:\!A_\gamma) &= -2C[\,\mathcal{P}_\oplus A_\gamma - (\mathcal{P}_\otimes + v_e\mathcal{P}_\oplus)d'A_Z\,], \\ C(D_{VA}\!:\!A_\gamma) &= -C(\mathcal{P}_\oplus + v_e\mathcal{P}_\otimes)d'B_Z, \end{aligned}$$

$$
\begin{aligned}
&C(E_{VA}\!:\!A_\gamma) = C(\mathcal{P}_\otimes + v_e\mathcal{P}_\oplus)d' B_Z,\\
&C(D_V\!:\!A_Z) = -2C[\,(\mathcal{P}_\oplus + v_e\mathcal{P}_\otimes)d' A_\gamma - \{2v_e\mathcal{P}_\oplus + (1+v_e^2)\mathcal{P}_\otimes\}d'^2 A_Z\,],\\
&C(E_V\!:\!A_Z) = 2C[\,(\mathcal{P}_\otimes + v_e\mathcal{P}_\oplus)d' A_\gamma - \{2v_e\mathcal{P}_\otimes + (1+v_e^2)\mathcal{P}_\oplus\}d'^2 A_Z\,],\\
&C(D_{VA}\!:\!A_Z) = C[\,2v_e\mathcal{P}_\oplus + (1+v_e^2)\mathcal{P}_\otimes\,]d'^2 B_Z,\\
&C(E_{VA}\!:\!A_Z) = -C[\,2v_e\mathcal{P}_\otimes + (1+v_e^2)\mathcal{P}_\oplus\,]d'^2 B_Z,
\end{aligned}
\tag{A.15}
$$

where $v_e = -1+4\sin^2\theta_W$, $d' \equiv s/[4\sin\theta_W\cos\theta_W(s-M_Z^2)]$, two polarization factors $\mathcal{P}_\oplus$ and $\mathcal{P}_\otimes$ are defined as

$$
\mathcal{P}_\oplus \equiv P_{e^-} + P_{e^+}, \qquad \mathcal{P}_\otimes \equiv 1 + P_{e^-}P_{e^+},
$$

and the others are thereby given as

$$
\begin{aligned}
&C(D_A\!:\!B_v) = 2C(D_{VA}\!:\!A_v), && C(E_A\!:\!B_v) = 2C(E_{VA}\!:\!A_v),\\
&C(D_{VA}\!:\!B_v) = C(D_V\!:\!A_v)/2, && C(E_{VA}\!:\!B_v) = C(E_V\!:\!A_v)/2,\\
&C(G_1\!:\!C_v) = C(D_V\!:\!A_v)/2, && C(G_2\!:\!C_v) = C(E_V\!:\!A_v)/2,\\
&C(G_3\!:\!C_v) = C(D_{VA}\!:\!A_v), && C(G_4\!:\!C_v) = C(E_{VA}\!:\!A_v),\\
&C(F_1\!:\!D_v) = -C(D_V\!:\!A_v)/2, && C(F_2\!:\!D_v) = -C(E_V\!:\!A_v)/2,\\
&C(F_3\!:\!D_v) = -C(D_{VA}\!:\!A_v), && C(F_4\!:\!D_v) = -C(E_{VA}\!:\!A_v).
\end{aligned}
\tag{A.16}
$$

As explained in the main text, they are not always independent of each other. When $P_\oplus = \pm P_\otimes$, i.e., $P_{e^-} = P_{e^+} = \pm 1$, we have

$$
C(\{D_V, E_V, D_{VA}, E_{VA}\}\!:\!A_Z) = \mp(1\pm v_e)d' C(\{D_V, E_V, D_{VA}, E_{VA}\}\!:\!A_\gamma),
$$

As a consequence of the above relations one gets

$$
\mathcal{F}^{f(*)}_{\{A,B,C,D\}Z}(x,\theta) = \mp(1\pm v_e)d'\mathcal{F}^{f(*)}_{\{A,B,C,D\}\gamma}(x,\theta).
$$

In this case all we can determine (for the production form factors) are the following four combinations

$$
\mathrm{Re}(\delta\{A,B,C,D\}_\gamma \mp (1\pm v_e)d'\delta\{A,B,C,D\}_Z).
$$

Finally we present here formulas for $D^{(*)}_{V,A,VA}$, $E^{(*)}_{V,A,VA}$, $F^{(*)}_{1\sim 4}$, $G^{(*)}_{1\sim 4}$ for completeness:

$$
\begin{aligned}
D^{(*)}_{V,\,A,\,VA} &= \mathcal{P}_{\otimes} D_{V,\,A,\,VA} - \mathcal{P}_{\oplus} E_{V,\,A,\,VA}, \\
E^{(*)}_{V,\,A,\,VA} &= \mathcal{P}_{\otimes} E_{V,\,A,\,VA} - \mathcal{P}_{\oplus} D_{V,\,A,\,VA}, \\
F^{(*)}_{1,\,2,\,3,\,4} &= \mathcal{P}_{\otimes} F_{1,\,2,\,3,\,4} - \mathcal{P}_{\oplus} F_{2,\,1,\,4,\,3}, \\
G^{(*)}_{1,\,2,\,3,\,4} &= \mathcal{P}_{\otimes} G_{1,\,2,\,3,\,4} - \mathcal{P}_{\oplus} G_{2,\,1,\,4,\,3},
\end{aligned}
\tag{A.17}
$$

for

$$
\begin{aligned}
D_V \;&\equiv\; C\,[\,A_\gamma^2 - 2A_\gamma A_Z v_e d' + A_Z^2(1+v_e^2)d'^2 + 2(A_\gamma - A_Z v_e d')\mathrm{Re}(\delta A_\gamma) \\
&\qquad -2\{A_\gamma v_e d' - A_Z(1+v_e^2)d'^2\}\mathrm{Re}(\delta A_Z)\,], \\
D_A \;&\equiv\; C\,[\,B_Z^2(1+v_e^2)d'^2 - 2B_Z v_e d'\mathrm{Re}(\delta B_\gamma) + 2B_Z(1+v_e^2)d'^2\mathrm{Re}(\delta B_Z)\,], \\
D_{VA} \;&\equiv\; C\,[\,-A_\gamma B_Z v_e d' + A_Z B_Z(1+v_e^2)d'^2 - B_Z v_e d'(\delta A_\gamma)^* \\
&\qquad +(A_\gamma - v_e d' A_Z)\delta B_\gamma + B_Z(1+v_e^2)d'^2(\delta A_Z)^* \\
&\qquad -\{A_\gamma v_e d' - A_Z(1+v_e^2)d'^2\}\delta B_Z\,], \\
E_V \;&\equiv\; 2C\,[\,A_\gamma A_Z d' - A_Z^2 v_e d'^2 + A_Z d'\mathrm{Re}(\delta A_\gamma) + (A_\gamma d' - 2A_Z v_e d'^2)\mathrm{Re}(\delta A_Z)\,], \\
E_A \;&\equiv\; 2C\,[\,-B_Z^2 v_e d'^2 + B_Z d'\mathrm{Re}(\delta B_\gamma) - 2B_Z v_e d'^2\mathrm{Re}(\delta B_Z)\,], \\
E_{VA} \;&\equiv\; C\,[\,A_\gamma B_Z d' - 2A_Z B_Z v_e d'^2 + B_Z d'(\delta A_\gamma)^* + A_Z d'\delta B_\gamma \\
&\qquad -2B_Z v_e d'^2(\delta A_Z)^* + (A_\gamma d' - 2A_Z v_e d'^2)\delta B_Z\,], \\
F_1 \;&\equiv\; C\,[\,-(A_\gamma - A_Z v_e d')\delta D_\gamma + \{A_\gamma v_e d' - A_Z(1+v_e^2)d'^2\}\delta D_Z\,], \\
F_2 \;&\equiv\; C\,[\,-A_Z d'\delta D_\gamma - (A_\gamma d' - 2A_Z v_e d'^2)\delta D_Z\,], \\
F_3 \;&\equiv\; C\,[\,B_Z v_e d'\delta D_\gamma - B_Z(1+v_e^2)d'^2\delta D_Z\,], \\
F_4 \;&\equiv\; C\,[\,-B_Z d'\delta D_\gamma + 2B_Z v_e d'^2\delta D_Z\,], \\
G_1 \;&\equiv\; C\,[\,(A_\gamma - A_Z v_e d')\delta C_\gamma - \{A_\gamma v_e d' - A_Z(1+v_e^2)d'^2\}\delta C_Z\,], \\
G_2 \;&\equiv\; C\,[\,A_Z d'\delta C_\gamma + (A_\gamma d' - 2A_Z v_e d'^2)\delta C_Z\,], \\
G_3 \;&\equiv\; C\,[\,-B_Z v_e d'\delta C_\gamma + B_Z(1+v_e^2)d'^2\delta C_Z\,], \\
G_4 \;&\equiv\; C\,[\,B_Z d'\delta C_\gamma - 2B_Z v_e d'^2\delta C_Z\,]
\end{aligned}
\tag{A.18}
$$

with $C \equiv 1/(4\sin^2\theta_W)$.

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